Bootstrap P-value Computation

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Mplus computes three types of bootstrap confidence intervals: symmetric, non-symmetric and bias-corrected. We describe these methods below and we also discuss how to obtain the corresponding P-values. Mplus currently computes the P-value only for the symmetric method, and that is what can be found in the Mplus output (regardless of which type of confidence interval is requested). The purpose of this note is to describe how one can obtain the P-values corresponding to the other two methods with the information available in Mplus 8.8. This is particularly useful when the confidence interval (non-symmetric or bias-corrected) leads to one significance conclusion while the P-value (from the symmetric method) produces another significance conclusion. The contradiction is due to the fact that the P-value reported in the Mplus output always uses the symmetric method, while the confidence interval might use a different method. In such contradictory situations, it is recommended to use the significance conclusion that is based on the confidence interval. If in addition, a P-value is needed to support the result of the non-symmetric/bias-corrected confidence interval, the manual method described below for computing such a P-value can be utilized. The discussion here applies to both the maximum-likelihood (ML) estimation as well as the weighted least squares estimation (WLS/WLSMV/WLSM/ULSMV).

1 Symmetric

This confidence interval is computed in Mplus when the option OUTPUT: CINT; is specified. Suppose that B bootstrap samples are drawn from the original sample and suppose that x_1, ..., x_B are the parameter estimates obtained from these bootstrap samples for a particular model parameter x.

The bootstrap SE is computed as follows

$$SE = \sqrt{\frac{B}{B-1}\left(\frac{1}{B}\sum_{i=1}^{B}x_i^2 - \left(\frac{1}{B}\sum_{i=1}^{B}x_i\right)^2\right)} = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}(x_i - \bar{x})^2}. \quad (1)$$

If the point estimate for this parameter is x_0, i.e., this is the estimate obtained from the original sample, then the 95% confidence interval is computed as

$$[x_0 - 1.96SE, x_0 + 1.96SE]. \quad (2)$$

In certain situations, reporting the confidence interval alone is not sufficient as that confidence interval only guarantees significance at the 95% level. It does not carry information about how strong the evidence is for the statistical significance. This can be remedied by also reporting the corresponding P-value. The P-value is the probability that the true parameter is actually 0 (even though the model estimate is not). A very small P-value, such as 0.001, would indicate a substantial statistical evidence that the parameter estimate is significant. The 95% confidence interval does not include the 0 value precisely when the P-value is smaller than 5%.

The P-value corresponding to the confidence interval (2) is computed as

$$2(1 - \Phi^{-1}(x_0/SE)), \quad (3)$$

if x_0 is positive and it is computed as

$$2\Phi^{-1}(x_0/SE), \quad (4)$$

if x_0 is negative, where Φ denotes the standard normal distribution function. These P-values are reported in the Mplus output file.

2   Non-symmetric

This confidence interval is computed in Mplus when the option OUTPUT: CINT(BOOT); is specified. Suppose that $x_{(1)} < x_{(2)} < ... < x_{(B)}$ are the ordered values of the parameter estimate across the bootstrap samples. The 95% confidence interval is computed as

$$[x_{(L)}, x_{(U)}] \quad (5)$$

where L is the nearest integer to $0.025B$ and U is the nearest integer to $0.975B$. The P-value corresponding to this interval can be computed as follows. Suppose that M is such that $x_{(M)} < 0 < x_{(M+1)}$, i.e., M of the values are negative and $B - M$ of the values are positive. If all the values are positive then M=0, and if all the values are negative then $M = B$. The two-sided P-value is

$$2 \cdot min(M/B, 1 - M/B). \qquad (6)$$

This P-value is currently not computed in Mplus. The output contains the symmetric interval P-value. It is possible however to manually compute this P-value using the following method.

The full bootstrap distribution must be saved as a data file using the command SAVEDATA: SAVE=BOOTSTRAP; RESULTS = file.dat. Import this file in an Excel or Google spreadsheet and sort the column that corresponds to the particular parameter. The number M is the number of negative values in the bootstrap distribution which is easy to get once the data is sorted in the spreadsheet. To improve the accuracy of M and the p-value, it is recommended that the number of bootstrap draws is at least 1000, but only if the p-value is near the critical level of 5%. A confidence interval for the p-value is easy to obtain as well. A simple rule that can be used is as follows. If the estimate for the p-value is P the confidence interval for P is $P \pm 2\sqrt{P(2 - P)/B}$. As a reference, with 1000 bootstrap draws the accuracy of the P-value estimate is $\pm 6\%$ and is worst for $P = 0.5$. With 1000 bootstrap draws, the accuracy of the P-value at $P = 0.05$ is $\pm 2\%$, that is, the confidence interval is $[0.03, 0.07]$.

3 Bias-corrected

This confidence interval is computed in Mplus when the option OUTPUT: CINT(BCBOOT); is specified. Let's assume that x_0 is positive and let N be such that $x_{(N)} < x_0 < x_{(N+1)}$, i.e., N out of the B bootstrap estimates are smaller than the point estimate x_0. We compute

$$z_0 = \Phi^{-1}(N/B) \qquad (7)$$

and this quantity is referred to as the bias-correction. If this quantity is 0, i.e. $N = B/2$, the bias-corrected confidence interval would be identical to the non-symmetric confidence interval.

Let L be the nearest integer to

$$B \cdot \Phi(2z_0 - 1.96) \tag{8}$$

and U be the nearest integer to

$$B \cdot \Phi(2z_0 + 1.96) \tag{9}$$

The 95% bias-corrected confidence interval is computed as

$$[x_{(L)}, x_{(U)}]. \tag{10}$$

The corresponding one-sided P-value is computed as

$$\Phi(-2z_0 + \Phi^{-1}(M/B)) = \Phi(-2\Phi^{-1}(N/B) + \Phi^{-1}(M/B)) \tag{11}$$

and to get the two-sided P-value we would multiply by 2. Statistical significance here is given when $M < L$ because that is precisely when the 0 value is below the confidence interval ($M + 1 \leq L$ and therefore $0 < x_{(M+1)} \leq x_{(L)}$). This P-value will be smaller than 0.05 precisely when the confidence interval does not include 0. That is because if $M < L$, then the one-sided P-value

$$\Phi(-2z_0 + \Phi^{-1}(M/B)) < \Phi(-2z_0 + \Phi^{-1}(L/B)) \approx \Phi(-1.96) \approx 0.025. \tag{12}$$

As in the case of the non-symmetric confidence interval, Mplus does not compute this P-value but the value can be obtained from the bootstrap distribution. The full bootstrap distribution must be saved as a data file using the command SAVEDATA: SAVE=BOOTSTRAP; RESULTS = file.dat. Import this file in an Excel or Google spreadsheet and sort the column that corresponds to the particular parameter. The number M is the number of negative values in the bootstrap distribution and the number N is the number of values that are smaller than the point estimate. Both number are easy to get once the data is sorted in the spreadsheet. A confidence interval for the P-value is more difficult to obtain here but one can use the approach described earlier for the non-symmetric method as an approximation.

If the point estimate x_0 is negative the one-sided P-value is computed similarly and is equal to

$$1 - \Phi(-2z_0 + \Phi^{-1}(M/B)) = 1 - \Phi(-2\Phi^{-1}(N/B) + \Phi^{-1}(M/B)) \tag{13}$$

while the two-sided P-value would multiply the above quantity by 2.